

St Michael's RCP School

Calculation Policy

Written Methods

- This policy contains the key pencil and paper procedures that are to be taught throughout the school. It has been written to ensure consistency and progression throughout the school.
- Although the main focus of this part of the policy is on pencil and paper procedures it is important to recognise that **the ability to calculate mentally lies at the heart of numeracy (refer to Year group lists of mental maths skills).**
- **Mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it.** In every written method there is an element of mental processing.
- Written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies. Rapid recall strategies and mental calculation methods will serve to reinforce and supplement written methods.
- The progression in development and understanding of methods is organised in stages including informal recording and jottings that help pupils to understand and use standard written methods.
- Children should not be discouraged from using previously taught methods with which they are secure, while the new concepts are becoming embedded. Teaching will continue to use a variety of resources, apparatus, models, images (including hundred squares), number tracks, number lines and empty number lines to explain and support calculations where appropriate, and encourage the use of jottings.

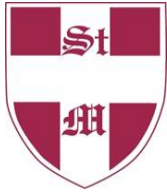
- The choice of method made by pupils will be discussed and challenged by adults in order to develop their mathematical thinking and reasoning.

- The long-term aim is for children to be able to select an efficient written method of their choice that is appropriate for a given task. They should do this by asking themselves:

- Can I do this in my head?

- Can I do this in my head using drawings or jottings?

- Do I need to use a written method?

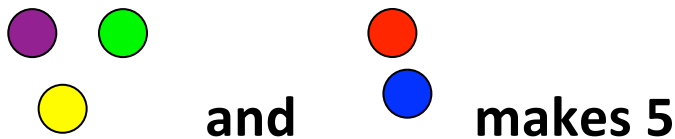


Addition

One key method will be taught and revisited throughout the school:

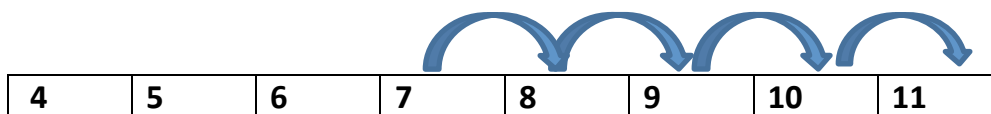
- Pupils will be taught to count on from most significant (larger) number
- Pupils will be taught partitioning
- This will be followed by addition of least significant (smaller) number to most significant (larger) number
- Pupils will be taught standard formal method of calculation (column addition)

Begin to relate addition to combining two groups of objects



Count along number tracks beginning with a larger number

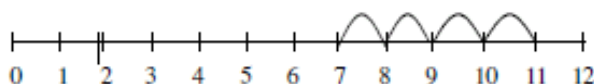
$$4 + 7 =$$



Use vertical number tracks too.

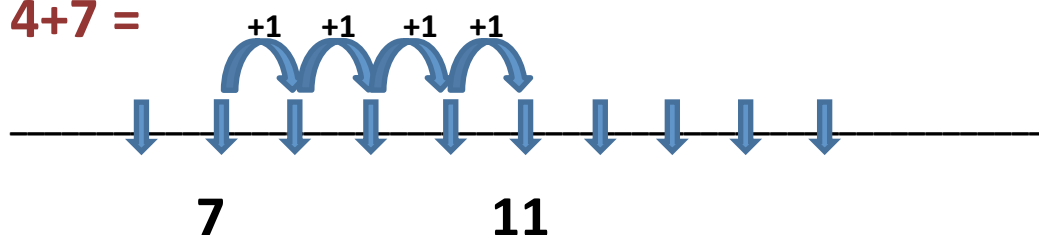
Count along numbered number lines beginning with a larger number

$$4 + 7 =$$



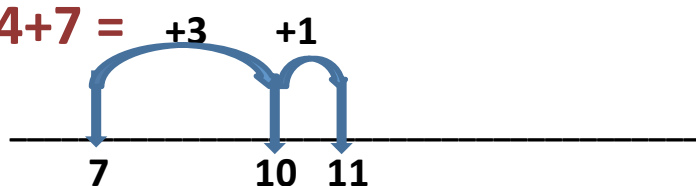
Use and count along number lines with missing numbers – begin with a larger number

$4 + 7 =$



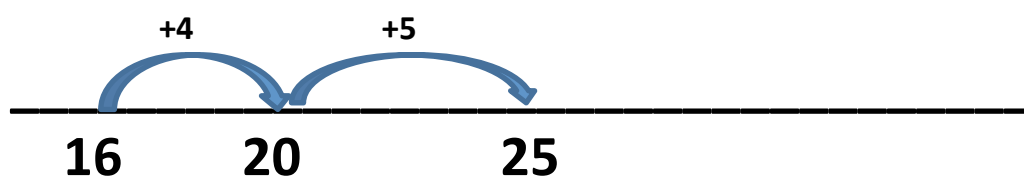
Draw own number lines - begin with a larger number (use number bonds)

$4 + 7 =$



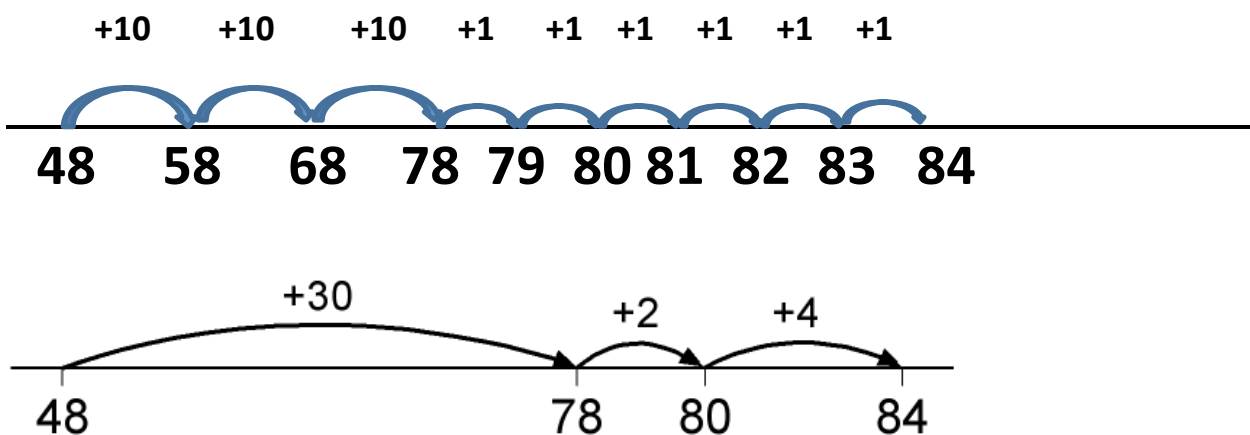
Draw own number lines – begin with a larger number and go to the next multiple of 10 (use number bonds)

$9 + 16 =$



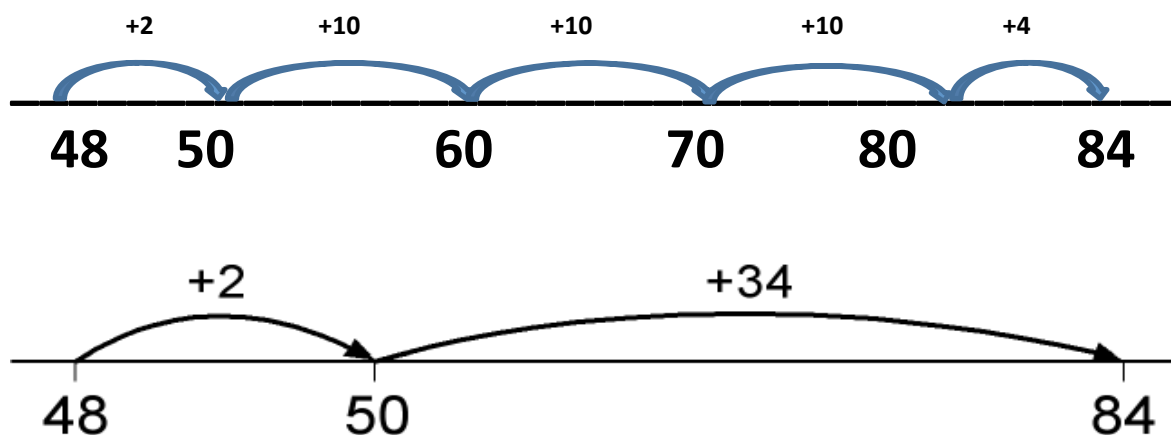
Draw own number lines – begin with a larger number and partition a smaller number into tens and units

$36 + 48 =$



Draw own number lines – begin with a larger number and go to the next multiple of 10 (use number bonds)

$$36 + 48 =$$



Put largest number first and **partition only** smaller number

$$47 + 76 =$$

$$56 + 129 =$$

$$118 + 256 =$$

$$76 + 40 = 116$$

$$129 + 50 = 179$$

$$256 + 100 = 356$$

$$116 + 7 = 123$$

$$179 + 6 = 185$$

$$356 + 10 = 366$$

$$366 + 8 = 374$$

Partition both numbers into tens and ones and recombine – this method leads directly into expanded column addition

$$47 + 76 =$$

$$40 + 70 = 110$$

$$7 + 6 = 13$$

$$110 + 13 = 123$$

Expanded column addition

$$47 + 76 =$$

$$\begin{array}{r} 40 + 7 \\ + 70 + 6 \\ \hline 110 + 13 = 123 \end{array}$$

$$375 + 288 =$$

$$\begin{array}{r} 300 + 70 + 5 \\ + 200 + 80 + 8 \\ \hline 500 + 150 + 13 = 663 \end{array}$$

Column addition - Progress to adding units first

$$\begin{array}{r} 47 + 76 = \\ + 76 \\ \hline 13 \quad (\text{as the total of 7 and 6}) \\ \underline{110} \quad (\text{as the total of 40 and 70}) \\ 123 \end{array}$$

$$\begin{array}{r} 358 + 73 = \\ + 73 \\ \hline 11 \quad (\text{as the total of 8 and 3}) \\ 120 \quad (\text{as the total of 50 and 70}) \\ \underline{300} \quad (\text{as the total of 300 and 0}) \\ 431 \end{array}$$

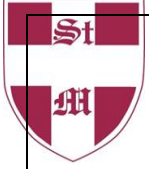
Lead to formal method (compact column method), showing numbers carried underneath:

$$\begin{array}{r} 358 + 73 = \\ + 73 \\ \hline 431 \\ 1 \end{array} \qquad \begin{array}{r} 3587 + 675 = \\ + 675 \\ \hline 4262 \\ 1 \end{array}$$

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.
Extend to decimals (same number of decimal places) and adding several numbers (with different numbers of digits).
Model negative numbers on a number line.

Extend to numbers with any number of digits and decimals with 1 and 2 decimal places (decimal point must be in line):

$$124.9 + 117.25 =$$



124.90 → *write in a zero to keep the place value*

+ 117.25

242.15

1 1

Subtraction

One key method will be taught and revisited throughout the school:

- Pupils will be taught to count back from most significant (larger) number
- Pupils will be taught partitioning numbers in different ways
- This will be followed by subtraction of least significant (smaller) number that is partitioned from most significant (larger) number
- Pupils will be taught standard formal method of calculation (column subtraction)
- It is crucial for children to understand subtraction not only as taking away or counting back from the larger number but also as a difference between numbers that can be worked out by counting on from the smaller number.

TAKE AWAY

Begin to relate subtraction to taking away

There are 6 dogs in a park and 3 of them run away. How many dogs are there left?

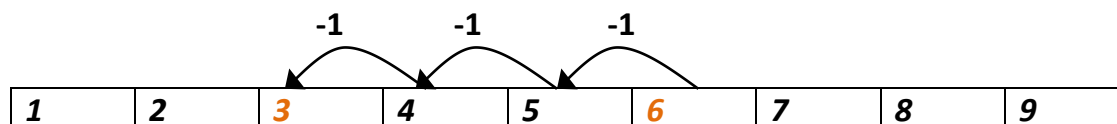


3 dogs are left

Count backwards in ones on number tracks beginning with a larger number

This should be one of the early stages in understanding subtraction as children can see that the number is getting smaller/less by counting back.

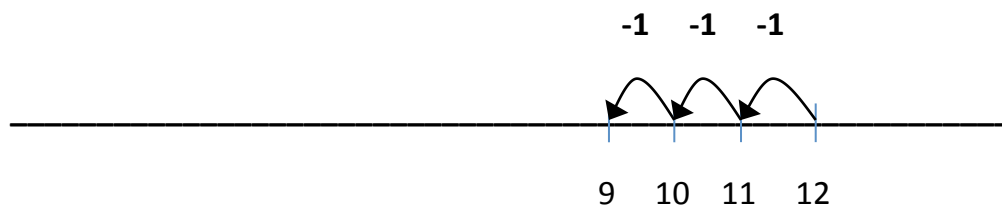
There are 6 dogs in a park and 3 of them run away. How many dogs are there left?



Counting backwards in ones using a number line beginning with the larger number

This should be one of the early stages in understanding subtraction as children can see that the number is getting smaller/less by counting back.

$12 - 3 =$



Counting back in tens and ones using a blank number line

The most significant number is positioned at the end of the number line.

The less significant number is partitioned into tens and ones.

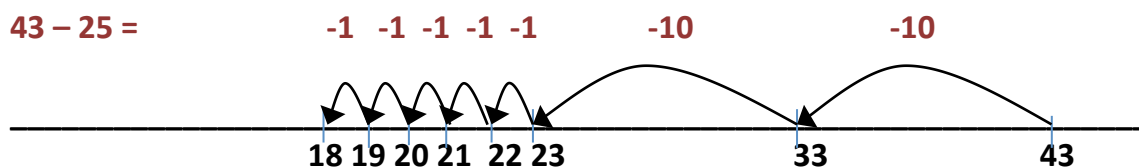
Count back the number of tens from the less significant number.

Label the numbers you land on after every jump.

Count back the ones from the less significant number.

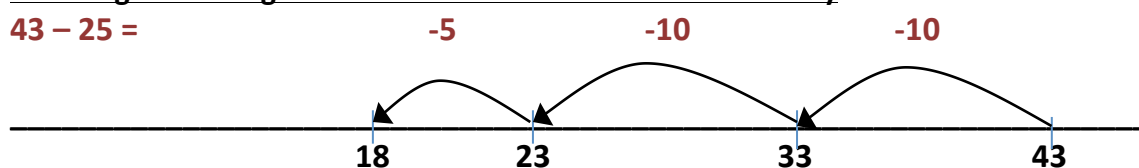
Label the numbers you land on after every jump.

$43 - 25 =$

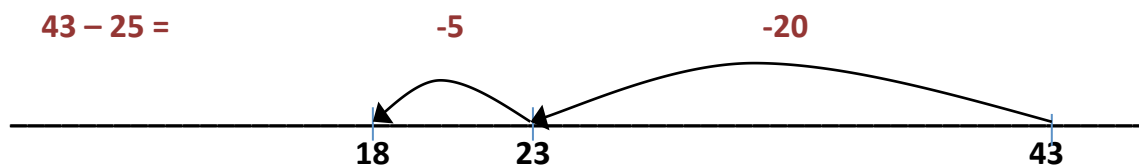


Counting back using a blank number line in more efficient way

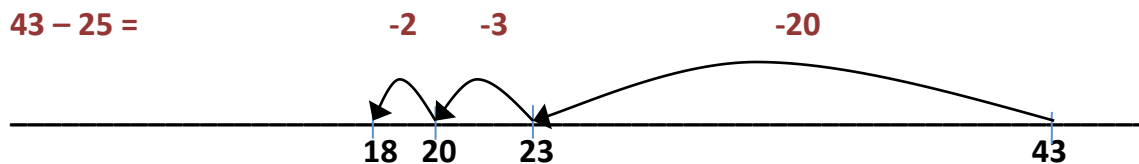
$43 - 25 =$



$43 - 25 =$



$43 - 25 =$



Partition **second number ONLY** – keep the larger number and partition only the less significant number

$73 - 46 =$



$73 - 40 = 33$

$33 - 6 = \underline{27}$



$123 - 76 =$



$123 - 70 = 53$

$53 - 6 = \underline{47}$



$$503 - 278 =$$

$$\begin{array}{ccc} \swarrow & \downarrow & \searrow \\ 200 & 70 & 8 \end{array}$$

$$503 - 200 = 303$$

$$303 - 70 = 233$$

$$233 - 8 = 225$$

$$\begin{array}{cc} \swarrow & \searrow \\ 3 & 5 \end{array}$$

Expanded column subtraction – Partitioning and decomposition

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support subtraction. It is important for the pupils to be able to **partition numbers in different ways**, not only into thousands, hundreds, tens and units. The expanded method enables children to see what happens to numbers in the standard written method.

Partitioning the number in hundreds, tens and units one under the other mirrors the column method.

$$76 - 43 =$$

$$\begin{array}{r} 70 + 6 \\ - 40 + 3 \\ \hline 30 + 3 = 33 \end{array}$$

$$73 - 46 =$$

$$\begin{array}{r} 60 \quad 13 \\ 70 + 3 \\ - 40 + 6 \\ \hline 20 + 7 = 27 \end{array}$$

$$503 - 278 =$$

$$\begin{array}{r} 400 \quad 90 \quad 13 \\ 500 + 0 + 3 \\ - 200 + 70 + 8 \\ \hline 200 + 20 + 5 = 225 \end{array}$$

$$741 - 367 =$$

$$\begin{array}{r} 600 \quad 130 \quad 11 \\ 700 + 40 + 1 \\ - 300 + 60 + 7 \\ \hline 300 + 70 + 4 = 374 \end{array}$$

Compact column subtraction

$$\begin{array}{r} 73 - 46 = \\ \quad \begin{array}{r} 6 \quad 13 \\ 73 \\ - 46 \\ \hline 27 \end{array} \end{array}$$

$$\begin{array}{r} 503 - 278 = \\ \quad \begin{array}{r} 4 \quad 9 \quad 13 \\ 503 \\ - 278 \\ \hline 225 \end{array} \end{array}$$

$$\begin{array}{r} 741 - 367 = \\ \quad \begin{array}{r} 6 \quad 13 \quad 11 \\ 741 \\ - 367 \\ \hline 374 \end{array} \end{array}$$

Extend to decimals ensuring that points line up under one another

FIND A DIFFERENCE

Children must be taught to understand subtraction not only as taking away or counting back from the larger number but also as a difference between numbers that can be worked out by counting on from the smaller number.

This model of subtraction will help pupils to solve problems like What is the difference between 7 and 11? or How much more do I need to save for a new pair of trainers if I have £25 and the trainers cost £43?

The 'difference' between numbers can be explained as 'How many jumps apart are X and Y?', 'How many more jumps do we have to make to get from X to Y?'. Bead strings and number lines can be used to model this.

Using practical equipment – What is the difference between these two numbers?

How many more green squares than orange square are there? How many fewer orange squares than green squares are there?



Compare two sets to find numerical difference

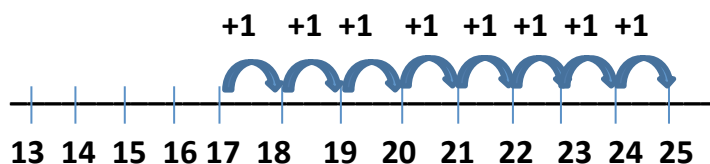
Using number tracks or number lines

What is the difference between 17 and 25?

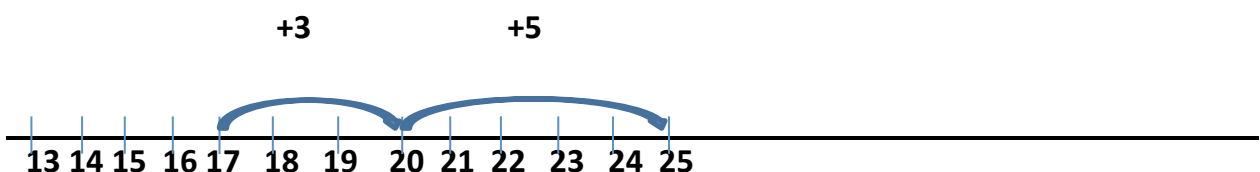
What is the difference between 25 and 17?

How many jumps are numbers 25 and 17 apart?

How many jumps are number 17 and 25 apart?



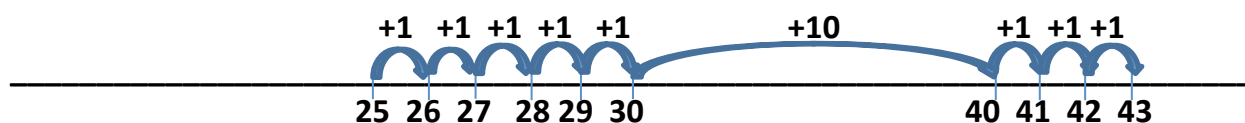
Use number bonds – jump to the next multiple of 10 or 100 when working with 3-digit numbers



Using blank number lines

The difference between 25 and 43

The difference is the total of all the jumps from 25 to 43 (18)

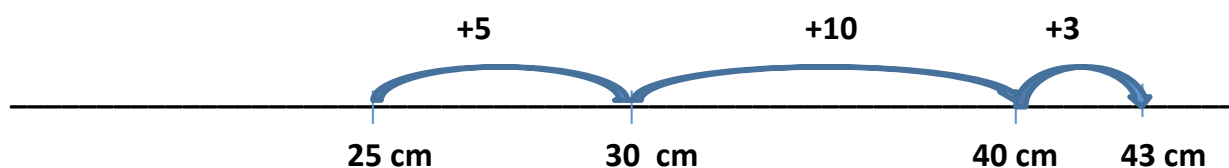


More efficient ways of finding the difference between numbers on a blank number line (use of number bonds)

The difference between 25 and 43

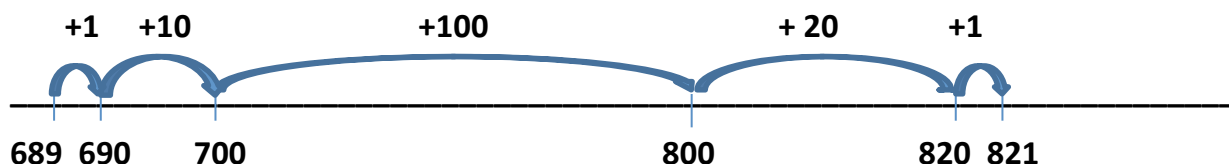
Jane's hair is 25 cm long and Susie's hair is 43 cm long. How much longer is Susie's hair than Jane's?

The difference is the total of all the jumps from 25 to 43 (18cm)



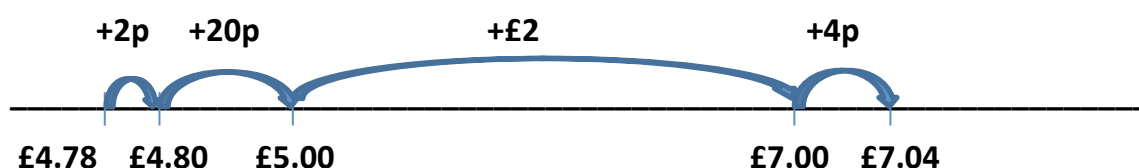
The difference between 689 and 821 is 132

The difference is the total of all the jumps from 689 to 821



The difference between £4.78 and £7.04 is £2.26

Peter has saved £4.78. He wants to buy a game for £7.04. How much more money does he need to save?





Multiplication

One key method will be taught and revisited throughout the school:

- Pupils will be taught to count on in jumps of a constant size
- Pupils will be taught partitioning that leads into a grid method - hundreds, tens and units are multiplied separately
- This is directly linked with the next stage in which pupils learn how to multiply in a column format, showing the working
- Pupils will be taught a standard formal method of multiplication with reduced recording of the working

Early stages of multiplication will focus on groups and sets, leading onto the learning multiplication tables facts up to 12×12 . Children should have a secure knowledge of all multiplication facts by the end of Year 4.

Equal groups/sets of objects



4 groups of 3 make 12

4 groups and there are 3 circles in each

Repeated addition

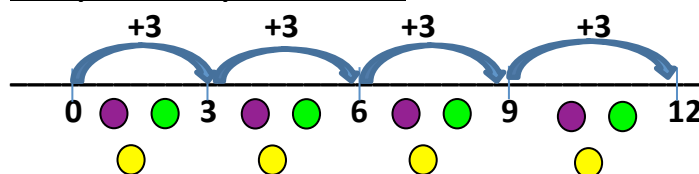
Pictorial representations are translated into number statements

$$3 + 3 + 3 + 3 = 12$$

$4 \times 3 = 12$ is same as 4 groups/sets/lots of 3

is same as number 3 repeatedly added 4 times

Repeated addition can be easily supported by equal jumps on a number line – make links with pictorial representations



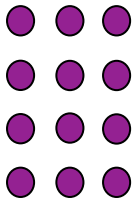
4 groups of 3

$$3 + 3 + 3 + 3 = 12$$

$$4 \times 3 = 12$$

Arrays – Commutativity of multiplication – two numbers can be multiplied in any order to give the same answer

4 groups of 3 can be presented in arrays – 4 groups of 3 give the same answer as 3 groups of 4



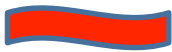
$3 \times 4 = 12$ (columns \rightarrow 3 groups of 4 or 3 columns of 4)

$4 \times 3 = 12$

(rows \rightarrow 4 groups of 3 or 4 rows of 3)

Scaling

I have a red ribbon that is 3cm long. Find a ribbon that is 4 times as long as the red ribbon.



3cm



$4 \times 3\text{cm} = 12\text{cm}$

Grid Method - a secure knowledge of place value and quick mental recall of multiplication facts are required.

It is important to model lining up thousands, hundreds, tens and units under each other.

A 2-digit number multiplied by a 1-digit number

$14 \times 6 =$

x	10	4
6	60	24

$60 + 24 = 84$ or

$$\begin{array}{r} 60 \\ + 24 \\ \hline 84 \end{array}$$

A 3-digit number multiplied by a 1-digit number

$237 \times 4 =$

x	200	30	7
4	800	120	28

$800 + 120 + 28 = 948$ or

$$\begin{array}{r} 800 \\ + 120 \\ + 28 \\ \hline 948 \end{array}$$

A 2-digit number multiplied by a 2-digit number

$47 \times 36 =$

x	40	7
30	1200	210
6	240	42
		282

$1410 + 282 =$

$$\begin{array}{r} 1410 \\ + 282 \\ \hline 1692 \end{array}$$

Extend to a 3-digit number multiplied by a 2-digit number

Decimals with one decimal place multiplied by a 1-digit number

Decimal points must be lined up under each other.

$$\begin{array}{r} 3.4 \times 4 = \\ \begin{array}{|c|c|c|} \hline x & 3 & 0.4 \\ \hline 4 & 12 & 1.6 \\ \hline \end{array} \end{array} \quad \begin{array}{r} 12.0 \\ + 1.6 \\ \hline 13.6 \end{array}$$

Decimals with one decimal point multiplied by a decimal with one decimal place

Decimal points must be lined up under each other.

$$\begin{array}{r} 7.2 \times 3.8 = \\ \begin{array}{|c|c|c|c|} \hline x & 7 & 0.2 & \\ \hline 3 & 21 & 0.6 & 21.6 \\ \hline 0.8 & 5.6 & 0.16 & 5.76 \\ \hline \end{array} \end{array} \quad \begin{array}{r} 21.60 \\ + 5.76 \\ \hline 27.36 \\ 1 \end{array}$$

Expanded short multiplication

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method.

A 2-digit number multiplied by a 1-digit number

$$\begin{array}{r} 34 \times 6 = \\ \begin{array}{r} 34 \\ \times 6 \\ \hline 24 \quad (6 \times 4) \\ 180 \quad (6 \times 30) \\ \hline 204 \\ 1 \end{array} \end{array}$$

A 3-digit number multiplied by a 1-digit number

$$\begin{array}{r} 237 \times 4 = \\ \begin{array}{r} 237 \\ \times 4 \\ \hline 28 \quad (4 \times 7) \\ 120 \quad (4 \times 30) \\ 800 \quad (4 \times 200) \\ \hline 948 \end{array} \end{array}$$

A 2-digit number multiplied by a 2-digit number

(any 'carry' digits are recorded below the line)

$$\begin{array}{r} 33 \\ \times 27 \\ \hline 21 \quad (7 \times 3) \\ 210 \quad (7 \times 30) \\ 60 \quad (20 \times 3) \\ 600 \quad (20 \times 30) \\ \hline 891 \end{array}$$

Compact short multiplication – when multiplying a 2-digit or a 3-digit number by a 1-digit number

The recording is reduced further, with 'carry' digits recorded below the line.

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ 5 \end{array}$$
$$\begin{array}{r} 125 \\ \times 7 \\ \hline 875 \\ 13 \end{array}$$

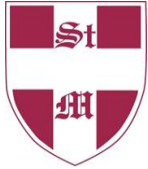
Expanded long multiplication – when multiplying a 2-digit or a 3-digit number by a 2-digit number

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \quad (7 \times 6) \\ 350 \quad (7 \times 50) \\ 120 \quad (20 \times 6) \\ \hline 1000 \quad (20 \times 50) \\ 1512 \\ 1 \end{array}$$
$$\begin{array}{r} 286 \\ \times 14 \\ \hline 24 \quad (4 \times 6) \\ 320 \quad (4 \times 80) \\ 800 \quad (4 \times 200) \\ 60 \quad (10 \times 6) \\ 800 \quad (10 \times 80) \\ 2000 \quad (10 \times 200) \\ \hline 4004 \\ 21 \end{array}$$

Compact long multiplication – 'carry' digits for multiplication are recorded above the columns, 'carry' digits for addition are recorded below the line

$$\begin{array}{r} 23 \\ \times 11 \\ \hline 23 \\ 230 \\ \hline 253 \end{array}$$
$$\begin{array}{r} 1 \\ 4 \\ 56 \\ \times 27 \\ \hline 392 \\ 1120 \\ \hline 1512 \\ 1 \end{array}$$
$$\begin{array}{r} 1 \\ -12 \\ 236 \\ \times 24 \\ \hline 944 \\ 4720 \\ \hline 5664 \\ 1 \end{array}$$

write in 0 as a place value holder



Division

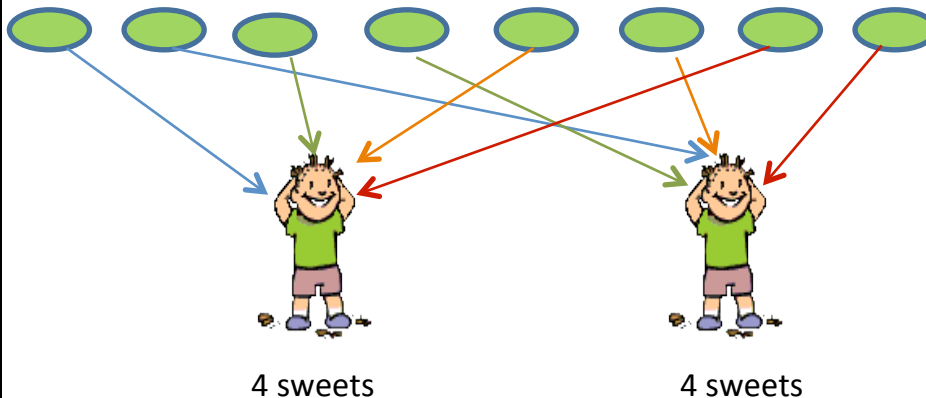
One key method will be taught and revisited throughout the school:

- Pupils will be taught to count back and on in jumps of a constant size
- They will also be taught to subtract multiples of a divisor
- Pupils will be taught a standard formal method of division showing their working
- This will lead into a standard formal method with reduced recording of the working
- It is crucial for children to understand division not only as sharing equally between a number of groups but also as a grouping
- Early stages of division will focus on equal sharing, grouping and making equal sets. Children should have a secure knowledge of counting on and back in 2s, 3s, 4s, 5s and 10s and they should recall halves and doubles of numbers to 20.
- Links between multiplication and division as inverse operations are crucial for pupils to progress to the formal division method.

Share objects equally between groups in a practical context – sharing one by one until all

objects are shared out – we know the number of groups we are sharing objects between

You have 8 sweets and you share them equally between 2 friends? How many sweets will each friend get?



Each friend will get 4 sweets.

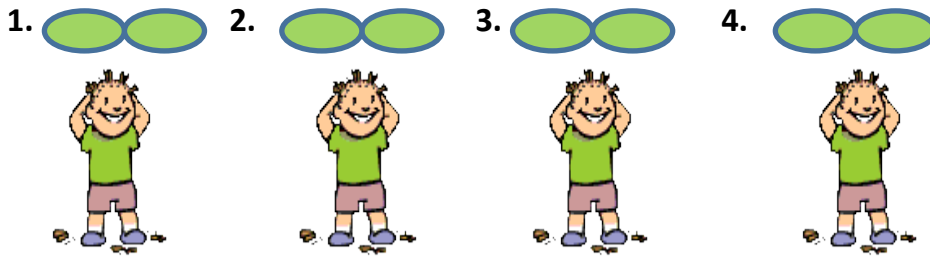
Grouping = repeated subtraction - we do not know the number of groups but we know how

many objects make a group/in each group -

I have 8 sweets and I give 2 sweets to each of my friends. How many friends do I have?

1. Take away two sweets and give them to the first friend.(1st group)
2. Take away two sweets and give them to your second friend. (2nd group)
3. Take away two sweets and give them to your third friend. (3rd group)

4. Take away the last two sweets and give them to your fourth friend. (4th group)



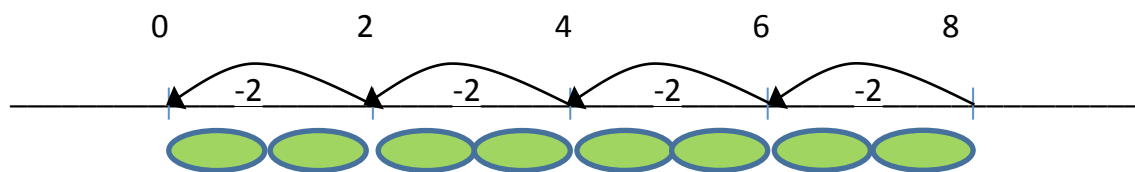
I have 4 friends.

With grouping model of division, questions like these should be used:

- How many 2s make 8?
- How many groups of 2 can you make out of 8?
- How many groups of 2 make 8?

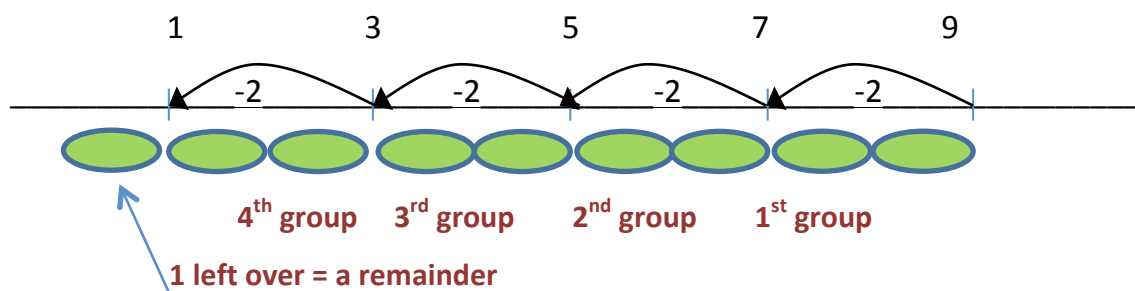
Grouping - Represent 'groups' for division on a number line using apparatus alongside the line - remainders can be introduced.

8 divided into groups of 2 – we know how many objects are in each group but we don't know the number of groups



9 divided into groups of 2

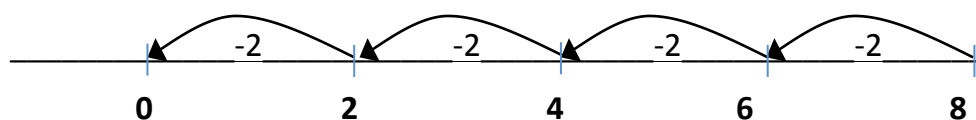
How many groups of 2 are there in 9? How many groups of 2 make 9?



Grouping on a blank number line

8 divided into groups of 2

$$8 : 2 = 4$$



Already at this stage, pupils should be encouraged to use their developing times table knowledge and, when working mentally, to count up in groups.

Use of repeated subtraction to subtract multiples of the divisor – quick mental recall of multiplication facts is crucial here

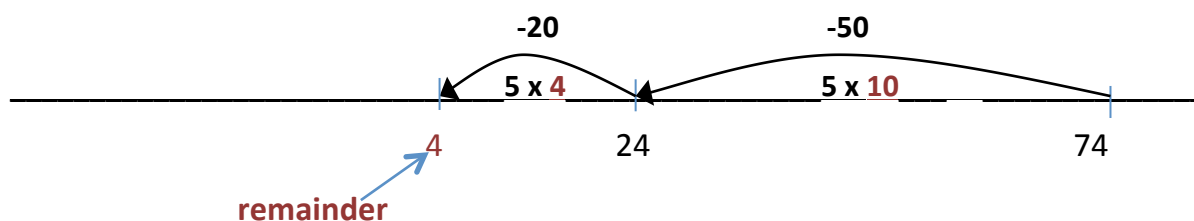
$$\begin{array}{r}
 74 : 5 = \quad 74 \\
 \quad \quad - 50 \quad (5 \times 10) \\
 \hline
 \text{divisor} \quad 24 \\
 \quad \quad - 20 \quad (5 \times 4) \\
 \hline
 \quad \quad \quad 4 \quad \leftarrow \text{remainder}
 \end{array}$$

Annotations: "first record the divisor" points to 24. "remainder" points to 4.

$10 + 4 = 14$

Answer: 14 remainder 4 or 14 r 4

This repeated subtraction can be supported by a number line.



Subtract larger multiples of the divisor

$$\begin{array}{r}
 256 : 7 = \quad 256 \\
 \quad \quad - 140 \quad (7 \times 20) \\
 \hline
 \text{divisor} \quad 116 \\
 \quad \quad - 70 \quad (7 \times 10) \\
 \hline
 \quad \quad \quad 46 \\
 \quad \quad - 42 \quad (7 \times 6) \\
 \hline
 \quad \quad \quad \quad 4 \quad \leftarrow \text{Remainder}
 \end{array}$$

Annotations: "first record the divisor" points to 116. "Remainder" points to 4.

$20 + 10 + 6 = 36$

Answer: 36 remainder 4

Standard written method - Long division

$$\begin{array}{r}
 \underline{25 \text{ r}3} \quad \leftarrow \text{answer} \\
 5 \overline{) 128} \\
 \underline{- 100} \quad (5 \times 20) \\
 \quad \quad 28 \\
 \underline{- 25} \quad (5 \times 5) \\
 \quad \quad \quad 3 \quad \rightarrow \text{remainder}
 \end{array}$$

Annotations: "divisor" points to 5. "remainder" points to 3.

Answer : 25 remainder 3

Less confident pupils can make simplified subtractions (instead of 5 x 20, they can subtract 5 x 10 twice)

Standard written method – short division - this method can be used when pupils are confident to divide 2-digit and 3-digit numbers by a single digit.

$$\begin{array}{r} \underline{25 \text{ r } 3} \\ 5 \overline{) 128} \\ \underline{28} \end{array} \quad \text{or} \quad \begin{array}{r} \underline{25 \text{ r } 3} \\ 5 \overline{) 12^2 8} \end{array}$$

Move on to calculating remainders as a fraction or decimal of a divisor.

Answer to the previous question would be recorded 25 $\frac{3}{5}$ or 25.6